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METHOD FOR CALCULATING THE SPECTRUM OF SCATTERED PHOTONS UNDER CONDITIONS OF AERIAL γ-MAPPING

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An analytical method for calculating the spectrum of multiply scattered γ-radiation of radioactive isotopes distributed in soil is proposed. Expressions for intensities of fluxes of photons scattered n times are obtained in the form of sums of multiple integrals of a simple form. The integration is performed numerically. Results of calculations of test examples are presented.

1. Introduction. In carrying out aerial γ-mapping of territories, for example, adjoining the zone of the Chernobyl accident, photons with primary energies of 0.1 to 2.0 MeV are mainly detected. These γ-quanta are the decay product of most active radionuclides (cesium-137, cesium-134, etc.) [1, 2].

Propagation of photons of a specified energy in the direction of the detector through layers of soil and air is characterized to good accuracy solely by Compton scattering of γ-quanta on atomic electrons. The integral cross sections of the two other main processes of interaction of γ-radiation with matter – the photoeffect and production of electron-positron pairs – are an order of magnitude smaller for silicates and aluminates, as well as for nitrogen and oxygen [2]. Therefore, evaluation of the spectrum of multiply scattered photons plays a crucial role in processing and interpreting results of aerial γ-mapping.

It is known that the two extreme approaches to the calculation of the spectrum of multiply scattered γ-quanta (numerical solution of the general transport equation and Monte Carlo simulation) are not sufficiently suitable for practical solution of the direct problem of scattering [3]. Therefore, based on various physical models, analytical methods of solution of particular problems of γ-quanta propagation through matter are developed [2-4]. In what follows, within this framework, we propose another analytical method for calculating the spectrum of multiply scattered photons. With this method, integral equations are obtained for intensities of fluxes of photons scattered n times. The integral equations are solved numerically. For test examples results of calculations obtained by the method proposed are compared with results obtained by the Monte Carlo simulation method (as implemented in the EGS-4 software package, verified on the basis of experimental data [5]).

2. Calculation of Intensities of Fluxes of Radiation Scattered by Soil. In this section, processes of scattering of γ-quanta in the atmosphere are not taken into account, i.e., we consider the case where fluxes of γ-quanta propagating in two media (soil and air) interact only with atoms of soil components. Appropriate generalizations are carried out in what follows.

It should be noted that the condition of cylindrical symmetry is satisfied for problems of detection of γ-radiation emitted by radioactive isotopes distributed in soil by a detector located at the height L above the ground and having an effective recording area ΔS.

Now let us consider a soil layer of thickness H containing both γ-radiation sources and scatterers. The medium is represented as an amorphous ensemble of atoms. Sources of γ-quanta will be considered as point sources and will be assumed to emit monochromatic and isotropic radiation. It can be assumed that as regards the process of detection of photons, their distribution over the radial coordinate is uniform. This assumption also holds for scatterers, whose role is played by electrons of atoms of the soil.

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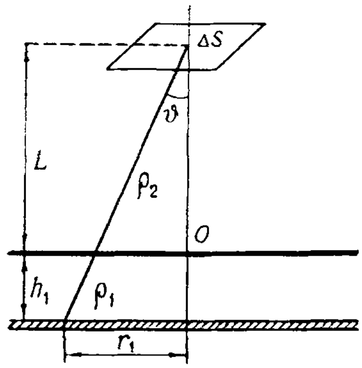

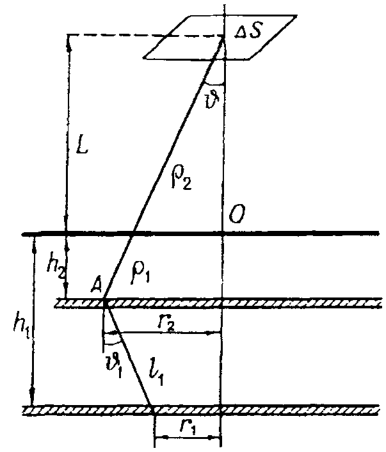

Fig. 1. Trajectory of a γ-quantum of direct radiation.

Fig. 2. Trajectory of a γ-quantum of singly scattered radiation.

We will follow the concept of the general method of successive collisions [4-6] and will sum fluxes of photons scattered different numbers of times. This approach, when applied to the solution of the transport equation, leads to a Neumann-type expansion corresponding to a sequence of integrated kernels of the equation. However, calculation of the resulting multiple integrals is complicated even for double scattering in the case of simple (point or plane) isotropic or collimated radiation sources due to the complicated form of the integrands. Therefore, based on general physical considerations, we write analytical expressions for the intensities $J^{(n)}$ of fluxes of γ-quanta that were scattered n times in the soil and hit the detector.

Let us represent a soil layer of thickness H as a set of infinite films of zero thickness. The radionuclide activity a and the concentration of scatterers N are assumed to be constant within each of these layers.

The number of γ-quanta of primary (unscattered) radiation with energy $\alpha_0 = \hbar\omega_0/mc^2$ that were emitted by a thin film located at the depth h_1 under the ground and then hit the detector can be represented (after multiplying by α_0) as follows:

$$J^{(0)}(\alpha_0, h_1) = \int_0^{2\pi}\int_0^{\infty} a(h_1)\,\alpha_0\, d\varphi_1\, r_1 dr_1\, \Delta S \cos\vartheta \frac{1}{4\pi(\rho_1+\rho_2)^2} \exp\left[-\mu_1(\alpha_0)\rho_1 - \mu_2(\alpha_0)\rho_2\right], \tag{1}$$

where μ_1 and μ_2 are the linear coefficients of radiation absorption for the soil and air, ρ_1 and ρ_2 are the paths of γ-quanta in the soil and air, r_1 and φ_1 are polar coordinates, and ϑ is the angle between the trajectory of the detected γ-quantum and the normal to the detector surface.

According to the geometry of the problem (Fig. 1) we have:

$$\cos\vartheta = \frac{L+h_1}{\rho_1+\rho_2} = \frac{L+h_1}{\sqrt{(L+h_1)^2 + r_1^2}};\quad \rho_1 = \frac{h_1}{\cos\vartheta};\quad \rho_2 = \frac{L}{\cos\vartheta}.$$

By integrating over the variable φ_1 and making the change of variables

$$u^2 = \frac{(L+h_1)^2 + r_1^2}{(L+h_1)^2},\quad udu = \frac{r_1 dr_1}{(L+h_1)^2},$$

with regard for these relationships we obtain

$$J^{(0)}(\alpha_0, h_1) = \frac{1}{2} a(h_1)\,\alpha_0\,\Delta S \int_0^{\infty} du \frac{\exp\left[(-\mu_1(\alpha_0)h_1 - \mu_2(\alpha_0)L)u\right]}{u^2}.$$

Upon the further change of variables

$$x = \frac{1}{u}, \quad dx = -\frac{1}{u^2}\,du,$$

we represent the expression for the intensity of the unscattered radiation in the final form

$$J^{(0)}(\alpha_0, h_1) = \frac{1}{2}\,a(h_1)\,\alpha_0\,\Delta S \int_0^1 dx \exp\left[-\frac{\mu_1(\alpha_0)\,h_1 + \mu_2(\alpha_0)\,L}{x}\right]. \tag{2}$$

To write the intensity of the singly scattered radiation being detected we turn to Fig. 2. The number of γ-quanta of the direct radiation from the thin film located at the depth h_1 hitting at an angle ϑ_1 the surface element $r_2 dr_2 d\varphi_2$ in the vicinity of the point A belonging to the thin film located at the depth h_2 is determined by expression (1). The probability $P(\alpha_0 \to \alpha_1)$ that a γ-quantum with the energy α_0 is scattered at the point A at an angle whose cosine equals w_1 and reduces its energy to the value α_1 is related to the differential scattering cross section $d\sigma$ within the limits of the solid angle $d\Omega$ and the energy interval $d\alpha$. It is expressed by the Klein–Nishina formula [4]

$$\frac{d\sigma(\alpha_0, \alpha_1, w_1)}{d\Omega\, d\alpha} = \frac{r_0^2}{2}\,\frac{1}{[1 + \alpha_1(1 - w_1)]^2}\left[1 + \frac{\alpha_0^2(1 - w_1)^2}{(1 + w_1^2)[1 + \alpha_0(1 - w_1)]}\right]\frac{\delta\left(1 + \frac{1}{\alpha_0} - \frac{1}{\alpha_1} - w_1\right)}{\alpha_1^2}; \tag{3}$$

where $r_0 = e^2/mc^2$ is the classical radius of an electron.

By summing up the contributions of all the scatterers of the thin film h_2 and then integrating over the complete set of these scattering films, we evaluate the intensity of the singly scattered radiation hitting the detector. Thus, the number of γ-quanta that were emitted by the thin film h_1 with the activity $a(h_1)$ and originally had the energy α_0 and then, upon scattering in soil with the concentration of scatterers $N(h_2)$, acquired the energy α_1 and then were detected is determined by the expression

$$J^{(1)}(\alpha_1, h_1) = \int_0^{2\pi}\int_0^{\infty} d\varphi_1\, r_1 dr_1\, a(h_1)\,\alpha_1\,\Delta S \cos\vartheta \times$$

$$\times \int_{-\infty}^{h_1} dh_2 \int_0^{2\pi}\int_0^{\infty} d\varphi_2\, r_2 dr_2\, N(h_2)\cos\vartheta_1\; W(\alpha_0 \to \alpha_1) \times$$

$$\times \frac{\exp[-\mu_1(\alpha_0)\,l_1]}{4\pi l_1^2}\,\frac{\exp[-\mu_1(\alpha_1)\rho_1 - \mu_2(\alpha_1)\rho_2]}{4\pi(\rho_1 + \rho_2)^2}. \tag{4}$$

The quantity $\cos\vartheta$ is expressed in terms of the lengths ρ_1 and ρ_2 of the paths traversed by the scattered γ-quantum in the soil and air:

$$\cos\vartheta = \frac{L + (h_1 - h_2)}{\rho_1 + \rho_2} = \frac{L + (h_1 - h_2)}{\sqrt{(L + (h_1 - h_2))^2 + r_2^2}};$$

$$\rho_1 = \frac{h_1 - h_2}{\cos\vartheta}; \quad \rho_2 = \frac{L}{\cos\vartheta}.$$

The quantity $\cos\vartheta_1$ is expressed in terms of the length l_1 of the path traversed by the primary γ-quantum in the soil prior to scattering:

$$\cos\vartheta_1 = \frac{|h_2|}{l_1}; \quad l_1 = \sqrt{h_2^2 + r_1^2}\,.$$

Expression (3) integrated over w_1 is used as $W(\alpha_0 \to \alpha_1)$. The relationship of this quantity to the probability of the above process is given by the formula

$$P(\alpha_0 \to \alpha_1) = \frac{W(\alpha_0 \to \alpha_1)}{4\pi l_1^2}\,.$$

The expressions for the intensities of the singly scattered and direct radiation turn out to have a similar structure. Therefore, upon integration of (4) over the angles φ_1 and φ_2 and after the change of the radial variables r_1 and r_2 performed similarly to the case of direct radiation, we finally obtain

$$J^{(1)}(\alpha_1, h_1) = \frac{1}{4}\,a(h_1)\,\alpha_1\,\Delta S\,W(\alpha_0 \to \alpha_1)\int\limits_{-\infty}^{h_1} dh_2\,N(h_2)\,\times$$

$$\times \int\limits_0^1 dx \exp\left[-\frac{\mu_1(\alpha_0)\,|h_2|}{x}\right]\int\limits_0^1 dy \exp\left[-\frac{\mu_1(\alpha_1)(h_1 - h_2) + \mu_2(\alpha_1)\,L}{y}\right]. \quad (5)$$

We come now to doubly scattered radiation. By using the concept of infinite films, we obtain the following picture of the process: a γ-quantum is emitted by the thin film h_1 and scatters at a point A belonging to the thin film h_2 by hitting the area element $r_2dr_2d\varphi_2$ at the angle ϑ_1, then it scatters again at the point B belonging to the thin film h_3 by hitting the area element $r_3dr_3d\varphi_3$ at the angle ϑ_2, and then it hits the detector. The expression for the intensity $J^{(2)}$ of the doubly scattered radiation being detected is as follows:

$$J^{(2)}(\alpha_2, h_1) = \int\limits_0^{2\pi}\int\limits_0^{\infty} d\varphi_1\,r_1dr_1\,a(h_1)\,\alpha_2\,\Delta S\cos\vartheta \int\limits_{-\infty}^{\infty} d\alpha_1\,\times$$

$$\times \int\limits_{-\infty}^{h_1} dh_2 \int\limits_0^{2\pi}\int\limits_0^{\infty} d\varphi_2\,r_2dr_2\,N(h_2)\cos\vartheta_1\,W(\alpha_0 \to \alpha_1)\,\times$$

$$\times \int\limits_{-\infty}^{h_1} dh_3 \int\limits_0^{2\pi}\int\limits_0^{\infty} d\varphi_3\,r_3dr_3\,N(h_2)\cos\vartheta_2\,W(\alpha_1 \to \alpha_2)\,\times$$

$$\times\,\frac{\exp[-\mu_1(\alpha_0)\,l_1]}{4\pi l_1^2}\,\frac{\exp[-\mu_1(\alpha_1)\,l_2]}{4\pi l_2^2}\,\frac{\exp[-\mu_1(\alpha_2)\,\rho_1 - \mu_2(\alpha_2)\,\rho_2]}{4\pi(\rho_1 + \rho_2)^2}\,. \quad (6)$$

The quantity $\cos\vartheta$ is expressed in terms of the lengths ρ_1 and ρ_2 of the paths traversed by the γ-quantum doubly scattered in the soil and air:

$$\cos\vartheta = \frac{L + (h_1 - h_3)}{\rho_1 + \rho_2} = \frac{L + (h_1 - h_3)}{\sqrt{(L + (h_1 - h_3))^2 + r_3^2}}\,;$$

$$\rho_1 = \frac{(h_1 - h_3)}{\cos\vartheta}; \quad \rho_2 = \frac{L}{\cos\vartheta}\,.$$

The quantity $\cos\vartheta_1$ is expressed in terms of the length l_1 of the path traversed by the primary γ-quantum in the soil prior to the first scattering:

$$\cos \vartheta_1 = \frac{|h_2|}{l_1}; \; l_1 = \sqrt{h_2^2 + r_1^2}.$$

The quantity $\cos \vartheta_2$ is expressed in terms of the length l_2 of the path traversed by the singly scattered γ-quantum in the soil prior to the second scattering:

$$\cos \vartheta_2 = \frac{|h_3 - h_2|}{l_2}; \; l_2 = \sqrt{|h_3 - h_2|^2 + r_2^2}.$$

Expression (3) integrated over w_2 is used as $W(\alpha_1 \to \alpha_2)$. This quantity is related to the probability of the above process by the formula

$$P(\alpha_1 \to \alpha_2) = \frac{W(\alpha_1 \to \alpha_2)}{4\pi l_2^2}.$$

Since no rigorous relationship between the initial and final energies of γ-quanta exists for multiple-scattering processes (in the case of single scattering, the presence of a single δ-function unambiguously relates α_0 and α_1 via the scattering angle), the integral over all intermediate values of α_1 appears in expression (6).

Upon integration in (6) over the angles φ_1, φ_2, φ_3 and after replacement of the radial variables r_1, r_2, r_3 by x, y, z, we obtain

$$J^{(2)}(\alpha_2, h_1) = \frac{1}{8} a(h_1)\, \alpha_2\, \Delta S \int_{-\infty}^{\infty} d\alpha_1\, W(\alpha_0 \to \alpha_1)\, W(\alpha_1 \to \alpha_2) \times$$

$$\times \int_{-\infty}^{h_1} dh_2\, N(h_2) \int_{-\infty}^{h_1} dh_3\, N(h_2) \int_0^1 dx \exp\left[-\frac{\mu_1(\alpha_0)\,|h_2|}{x}\right] \times$$

$$\times \int_0^1 dy \exp\left[-\frac{\mu_1(\alpha_1)\,|h_3 - h_2|}{y}\right] \int_0^1 dz \exp\left[-\frac{\mu_1(\alpha_2)(h_1 - h_3) + \mu_2(\alpha_2)\, L}{z}\right]. \tag{7}$$

In a similar manner, one can consider the process describing n successive scatterings of γ-quanta. The recursion formula for $J^{(n)}(\alpha_n, h_1)$ $(n = 2, 3, 4, \ldots)$ is as follows:

$$J^{(n)}(\alpha_n, h_1) = \int_0^{2\pi} \int_0^{\infty} d\varphi_1\, r_1 dr_1\, a(h_1)\, \alpha_n\, \Delta S \cos \vartheta \times$$

$$\times \int_{-\infty}^{\infty} d\alpha_1 \ldots \int_{-\infty}^{\infty} d\alpha_{n-1}\, W(\alpha_0 \to \alpha_1) \ldots W(\alpha_{n-1} \to \alpha_n) \times$$

$$\times \int_{-\infty}^{h_1} dh_2 \int_0^{2\pi} \int_0^{\infty} d\varphi_2\, r_2 dr_2\, N(h_2) \cos \vartheta_1 \ldots \times$$

$$\times \int_{-\infty}^{h_1} dh_n \int_0^{2\pi} \int_0^{\infty} d\varphi_n\, r_n dr_n\, N(h_2) \cos \vartheta_n \times$$

$$\times \frac{\exp[-\mu_1(\alpha_0)\, l_1]}{4\pi l_1^2} \ldots \frac{\exp[-\mu_1(\alpha_{n-1})\, l_n]}{4\pi l_n^2} \frac{\exp[-\mu_1(\alpha_n)\rho_1 - \mu_2(\alpha_n)\rho_2]}{4\pi(\rho_1 + \rho_2)^2} \tag{8}$$

or

$$J^{(n)}(\alpha_n, h_1) = \frac{1}{2^n} a(h_1)\,\alpha_n\,\Delta S \int_{-\infty}^{\infty} d\alpha_1 \ldots d\alpha_{n-1}\, W(\alpha_0 \to \alpha_1) \ldots W(\alpha_{n-1} \to \alpha_n) \times$$

$$\times \int_{-\infty}^{h_1} dh_2\, N(h_2) \ldots \int_{-\infty}^{h_1} dh_n\, N(h_2) \int_0^1 dx \exp\left[-\frac{\mu_1(\alpha_0)\,|h_2|}{x}\right] \times$$

$$\times \int_0^1 dy_1 \exp\left[-\frac{\mu_1(\alpha_1)\,|h_3 - h_2|}{y_1}\right] \ldots \int_0^1 dy_{n-1} \exp\left[-\frac{\mu_1(\alpha_{n-1})\,|h_{n+1} - h_n|}{y_{n-1}}\right] \times$$

$$\times \int_0^1 dz \exp\left[-\frac{\mu_1(\alpha_n)(h_1 - h_{n+1}) + \mu_2(\alpha_n) L}{z}\right]. \qquad (9)$$

Integration over intermediate energies of γ-quanta in multiple scattering and over cosines of scattering angles for evaluation of $W(\alpha_{n-1} \to \alpha_n)$ is carried out with account for the properties of the Dirac δ-function. In the case of single scattering the single integral over dw_1 is calculated simply according to the definition of the δ-function:

$$\int_{-1}^{1} dw_1\, F(w_1)\,\delta\left(1 + \frac{1}{\alpha_0} - \frac{1}{\alpha_1} - w_1\right) = F\left(1 + \frac{1}{\alpha_0} - \frac{1}{\alpha_1}\right).$$

In the expression for the intensity of doubly scattered radiation, a triple integral over dw_1, dw_2, and $d\alpha_1$ appears. For its transformation we use the rules of operations involving δ-function products and obtain

$$\int_{-1}^{1} dw_1 \int_{-1}^{1} dw_2 \int_{-\infty}^{\infty} d\alpha_1\, G(w_1, w_2)\,\delta\left(1 + \frac{1}{\alpha_0} - \frac{1}{\alpha_1} - w_1\right)\delta\left(1 + \frac{1}{\alpha_1} - \frac{1}{\alpha_2} - w_2\right) =$$

$$= \int_{-1}^{1} dw_1 \left[\frac{1}{1 + \frac{1}{\alpha_0} - w_1}\right]^2 G(w_1, \beta)\,(\theta(\beta + 1) - \theta(\beta - 1)),$$

where $\theta(\beta \pm 1)$ is the Heaviside θ-function, and $\beta \equiv 2 + 1/\alpha_0 - 1/\alpha_2 - w_1$. The presence of θ-functions in the expressions for the intensities of multiply scattered γ-quanta reflects the fact that contributions to the spectrum are made only by those particles for which the stepwise process of energy transformation $\alpha_0 \to \alpha_1 \to \ldots \to \alpha_n$ is allowed by the conservation laws.

Evaluation of $J^{(n)}$ requires calculation of multiple integrals, the number of which grows rapidly with n. However, the cumbersome structure of these expressions is easily transformed analytically into a sum of products of single integrals or a sum of multiple integrals of simple combinations of exponentials and polynomials. We illustrate this by the example of the expressions for the intensities of singly and doubly scattered γ-quanta.

Let us denote by $I^{(1)}$ the quantity equal to $J^{(1)}(\alpha_1, h_1)$ up to factors preceding the integration in Eq. (5). Then, by considering the integral over dh_2 within the limits from $-\infty$ to 0 and from 0 to h, one can easily obtain:

$$I^{(1)} = \int_0^{h_1} dh_2 f_1(h_2) f_2(h_1, h_2) + \int_0^1 x\,dx \int_0^1 y\,dy \frac{\exp\left[-\frac{\mu_1(\alpha_1) h_1 + \mu_2(\alpha_1) L}{y}\right]}{\mu_1(\alpha_0) y + \mu_1(\alpha_1) x},$$

$$f_1(h_2) = \int_0^1 dx \exp\left[-\frac{\mu_1(c_0) h_2}{x}\right\},$$

$$f_2(h_1, h_2) = \int_0^1 dy \exp\left[-\frac{\mu_1(\alpha_1)(h_1 - h_2) + \mu_2(\alpha_1) L}{y}\right]. \qquad (10)$$

Let the quantity $I^{(2)}$ be equal to $J^{(2)}(\alpha_2, h_2)$ accurate up to the factors preceding the integration over the variable h_2 in Eq. (7). We consider the integral over dh_2 within the limits from $-\infty$ to 0 and from 0 to h_1 and the integral over dh_3 within the limits from $-\infty$ to h_2 and from h_2 to h_1. It can be shown that

$$I^{(2)} = \int_0^{h_1} dh_2\, g_1(h_2) \int_{h_2}^{h_1} dh_3\, g_2(h_3, h_2)\, g_3(h_1, h_3) +$$

$$+ \int_0^{h_1} dh_2\, g_1(h_2)\, g_4(h_1, h_2) + \int_0^{h_1} dh_3\, g_5(h_1, h_3)\, g_6(h_3) +$$

$$+ \int_0^1 dx \int_0^1 dy \int_0^1 dz \left[\frac{\exp\left[-\dfrac{\mu_1(\alpha_2) h_1 + \mu_2(\alpha_2) L}{z}\right] z^2 xy}{(\mu_1(\alpha_1) z + \mu_1(\alpha_2) y)(\mu_1(\alpha_2) x + \mu_1(\alpha_0) z)} - \right.$$

$$\left. - \frac{\exp\left[-\dfrac{\mu_1(\alpha_2) h_1 + \mu_2(\alpha_2) L}{z}\right] x^2 zy}{(\mu_1(\alpha_1) x + \mu_1(\alpha_0) y)(\mu_1(\alpha_2) x + \mu_1(\alpha_0) z)} \right] . \quad (11)$$

$$g_1(h_2) = \int_0^1 dx \exp\left[-\frac{\mu_1(\alpha_0) h_2}{x}\right], \quad g_2(h_3, h_2) = \int_0^1 dy \exp\left[-\frac{\mu_1(\alpha_1)(h_3 - h_2)}{y}\right],$$

$$g_3(h_1, h_3) = \int_0^1 dz \exp\left[-\frac{\mu_1(\alpha_2)(h_1 - h_3) + \mu_2(\alpha_2) L}{z}\right],$$

$$g_4(h_1, h_2) = \int_0^1 dz \int_0^1 dy \frac{zy \exp\left[-\dfrac{\mu_1(\alpha_2)(h_1 - h_2) + \mu_2(\alpha_2) L}{z}\right]}{\mu_1(\alpha_1) z + \mu_1(\alpha_2) y},$$

$$g_5(h_1, h_3) = \int_0^1 dz \exp\left[-\frac{\mu_1(\alpha_2)(h_1 - h_3) + \mu_2(\alpha_2) L}{z}\right],$$

$$g_6(h_3) = \int_0^1 dx \int_0^1 dy \frac{xy \exp\left[-\dfrac{\mu_1(\alpha_1) h_3}{y}\right]}{\mu_1(\alpha_0) y + \mu_1(\alpha_1) x} .$$

To obtain the total spectrum of the γ-radiation from the soil layer where radionuclides are distributed according to the depth function $a(h_1)$, the corresponding expressions for $J^{(n)}(\alpha_n, h_1)$ should be integrated over h_1.

The description of the Compton scattering is universal for any medium, since the cross section of the Compton interaction with an electron does not depend on the effective atomic number of the medium, in contrast to the photoeffect and generation of pairs [3]. The linear coefficients of radiation absorption μ_1 and μ_2, which depend on the energy of the γ-quanta and the properties of the given medium, can be taken as tabulated data [1] or calculated by the formula

$$\mu(\alpha) = N\sigma_c(\alpha),$$

where σ_C is the integral cross section of Compton scattering of a photon on the electron when the effective atomic number of the medium and the mean concentration of atoms in it are known [4].

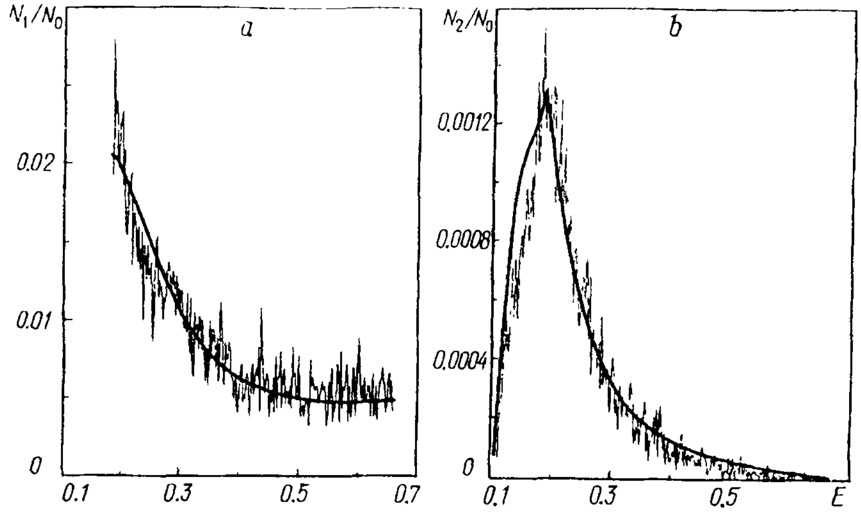

Fig. 3. Spectra of singly (a) and doubly (b) scattered radiation. E, MeV.

3. Generalization of the Results Obtained. The above-described procedure for calculating intensities of fluxes of radiation scattered in soil can easily be generalized by including processes of scattering of γ-quanta in air in the considerations. For example, the number of γ-quanta emitted by a radioactive film that are singly scattered in the atmosphere (the concentration of scatterers $N(\lambda)$, $\lambda \in [0, L]$) and hit the detector is determined by the equation

$$\tilde{J}^{(1)}(\alpha_1, h_1) = \frac{1}{4} a(h_1)\alpha_1 \Delta S\, W(\alpha_0 \to \alpha_1) \int_0^L d\lambda\, N(\lambda) \times$$

$$\times \int_0^1 dx \exp\left[-\frac{\mu_1(\alpha_0) h_1 + \mu_2(\alpha_0)\lambda}{x}\right] \int_0^1 dy \exp\left[-\frac{\mu_2(\alpha_1)(L-\lambda)}{y}\right].$$

It should also be noted that if one uses empirical or model analytical formulas for the cross sections of the photoeffect σ_{ph} and generation of electron–positron pairs σ_g, which depend strongly on the charge of the atomic nucleus of the particular chemical element, the total linear absorption coefficient can be calculated:

$$\mu'(\alpha) = N(\sigma_C + \sigma_{ph} + \sigma_g).$$

Then the method proposed is quite applicable to problems of aerial γ-mapping with account for all three main processes of interaction of γ-quanta with matter.

4. Results of Numerical Simulation. As an example, we present results provided by the above method of calculating the spectrum of scattered photons for a homogeneous cesium film with unit activity. The initial energy of the γ-quanta equals 0.662 MeV. Scattering takes place in a medium with an effective atomic number $Z = 13$ and a density $\rho = 1.42$ g/cm^3 (an analog of soddy-podzolic soil). The distribution of scatterers over the depth is considered to be uniform. For air, $Z = 7.5$, and $\rho = 1.29 \cdot 10^{-3}$ g/cm^3. The detecter is situated at a height $L = 30$ m above the ground and has unit detection area.

Figure 3 presents spectra of singly and doubly scattered radiation from the radioactive film located at a depth of 5 cm. Results of numerical integration of Eqs. (10) and (11) are represented by thick curves. Programs for a personal computer were written in FORTRAN. Thin curves present results obtained for the same example using the EGS-4 software package [5]. The numerical modeling has been performed for $N_0 = 10^7$ photons; here, the numbers N_1 and N_2 of particles singly and doubly scattered in the soil were calculated. It should be noted that the time required for solution of the problem is ten times longer when the EGS-4 package is used.

A comparison of data obtained by different methods allows one to state that the analytical method proposed leads to correct results.

5. **Conclusion.** The method proposed yields exact analytical expressions for detected intensities of fluxes of γ-quanta scattered n times. The method makes it possible to provide a fast solution to the direct problem of the theory of scattering for interpreting results of aerial γ-mapping of radioactively contaminated territories.

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